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Throughflow Effects on Thermal Convection in Variable Viscosity Ferromagnetic Liquids

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Abstract—The problem of thermal convection in temperature and magnetic field sensitive Newtonian ferromagnetic liquid is studied in the presence of uniform vertical magnetic field and throughflow. Using a combination of Galerkin and shooting techniques the critical eigenvalues are obtained for stationary mode. The effect of Prandtl number $(Pr > 1)$ on onset is insignificant and nonlinearity of non-buoyancy magnetic parameter M_3 is found to have no influence on the onset of ferroconvection. The magnetic buoyancy number, M_1 and variable viscosity parameter, V have destabilizing influences on the system. The effect of throughflow Peclet number, Pe is to delay the onset of ferroconvection and this effect is independent of the direction of flow.

Keywords—Ferroconvection, throughflow, temperature dependent viscosity, magnetic field dependent viscosity.

I. INTRODUCTION

FERROCONVECTION in a layer of ferromagnetic liquid plays a very important role in heat transfer problems. Finlayson [1] made a detailed study of thermal convection in a ferromagnetic liquid. The various aspects on the theory of thermoconvective instability in ferromagnetic liquids has recieved increasing importance over the years (see [2]-[11]).

The effect of different basic temperature gradients on the onset of ferroconvection driven by combined surface tension and buoyancy forces was provided by Shivakumara et al. [12]. Abraham and Siddheshwar [13] have examined the thermal instability in a layer of a ferromagnetic liquid when the boundaries are subjected to synchronous/asynchronous imposed time-periodic boundary temperatures (ITBT) and time-periodic body force (TBF) and showed that convection influence is controlled by (ITBT) and (TBF). The detailed study of linear and non-linear analyses using the generalized energy method for the convection problem in a ferromagnetic liquid with magnetic field dependent (MFD) viscosity was made by Sunil et al. [14], [15]. The significant contribution of ferromagnetic liquids in applications is provided by Rosensweig et al. [16] and Odenbach [17].

To suppress or augment the convection, the mechanisms that have been used effectively are Coriolis force due to rotation or external magnetic/electric fields or non-uniform temperature gradient across the liquid layer. A vertical throughflow has a significant influence on the stability of the system. The modified problem where the boundaries are permeable and there is injection of liquid through the upper plate and suction through the lower plate called the throughflow has been studied extensively by many authors (see [18]-[25]).

Now, we move on to the literature on thermal convection in liquids with variable viscosity. Siddheshwar et al. [26] studied the influence of an externally applied magnetic field on the Rayleigh-Bénard-Marangoni magnetoconvection with thermorheological effect in a Newtonian liquid for all possible boundary combinations. The detailed study on the onset of Rayleigh-Bénard, Bénard-Marangoni and Rayleigh-Bénard-Marangoni convections in a viscoelastic liquid with variable viscosity was provided by Sekhar and Jayalatha [27], [28]. Sekhar et al. [29] have studied the effects of magnetorheological and thermorheological parameters on Rayleigh-Bénard-Marangoni convection with non-uniform basic temperature gradient in ferromagnetic liquids and the influence of various parameters on the onset of convection has been analyzed. Various aspects of thermal convection in liquids with variable viscosity are studied extensively by many authors (see [30]-[32]).

In the paper, we study the effect of throughflow on the onset of thermal convection with temperature and magnetic field dependent viscosity in ferromagnetic liquids for all possible boundary combinations.

II. MATHEMATICAL FORMULATION

The physical configuration considered here consists of infinite, horizontal and variable viscosity ferromagnetic liquid layer of thickness d. The Cartesian co-ordinate system is taken with the lower plate in the xy-plane and z-axis vertically upwards. A constant throughflow of magnitude w_0 is maintained which is gravity alined or antigravity in its direction. The uniform magnetic field $H_i = (0, 0, H_0)$ is applied along the vertical direction. The lower and upper plates are maintained at constant temperatures $T_0 + \Delta T$ at z = 0 and T_0 at $z = d$ respectively (see Fig. 1).

A. Governing Equations

The governing equations which represent the above physical configuration are:

Continuity equation:

$$q_{i,i} = 0, \tag{1}$$

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TABLE I
NOMENCLATURE

a	dimensionless wave number	V	variable viscosity parameter
B_i	magnetic induction	w_0	constant vertical throughflow
C_{VH}	specific heat volume and magnetic field		**Greek symbols**
d	depth of the liquid layer	δ_T, δ_H	small positive constants
g_i	gravitational acceleration (0, 0,-g)	α	thermal expansion coefficient
H_i	components of applied magnetic field	Δ	difference of two values
H_0	applied magnetic field	κ	thermal conductivity
l, m	wave numbers	$\mu(H,T)$	variable viscosity
M_i	magnetization	μ_0	magnetic permeability
M_0	mean value of magnetization	ρ	density
M_1	buoyancy magnetic number	ρ_0	reference density
M_3	non-buoyancy magnetic number	ϕ	magnetic scalar potential
p	effective pressure	ω	frequency
Pe	throughflow Peclect number		**Subscripts and Superscripts:**
Pr	Prandtl number	b	basic state
q_i	components of velocty (u,v,w)	c	critical quantity
R	stationary Rayleigh number	0	reference value
t	time	$*$	dimensionless quantity
T	temperature		
T_0	constant temperature of the boundary	$'$	dimensional quantity

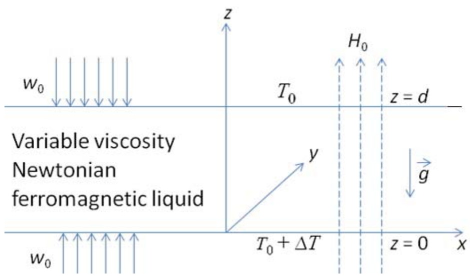

Fig. 1 Physical configuration of the problem

Momentum equation:

$$\rho_0\left(\frac{\partial q_i}{\partial t} + q_j q_{i,j}\right) = \mu_0(M_j H_{i,j}) + [\mu(H,T)(q_{i,j}+q_{j,i})]_{,i} - p_{,i} + \rho g_i, \tag{2}$$

Energy equation is assumed in the form:

$$\rho_0 C_{VH}\left(\frac{\partial T}{\partial t} + q_j T_{,j}\right) = \kappa T_{,jj}, \tag{3}$$

Density equation of state:

$$\rho = \rho_0[1-\alpha(T-T_0)], \tag{4}$$

Maxwell's equations:

$$B_{i,i} = 0 \quad \text{and} \quad \epsilon_{ijk}H_{k,j} = 0, \tag{5}$$

The magnetic induction equation given by

$$B_i = \mu_0(M_i + H_i), \tag{6}$$

Magnetic equation of state:

$$M = M_0 + \chi_m(H-H_0) + k_l(T-T_0). \tag{7}$$

The pyromagnetic coefficient and magnetic susceptibility are given by

$$k_l = -\left(\frac{\partial M}{\partial T}\right)_{H_0,T_0}, \qquad \chi_m = \left(\frac{\partial M}{\partial H}\right)_{H_0,T_0}.$$

Effective viscosity $\mu(H,T)$ is assumed as:

$$\mu(H,T) = \frac{\mu_0}{1+\delta_T(T-T_0)-\delta_H(H-H_0)}. \tag{8}$$

B. Basic State Solution

The solution in the quiescent basic state is given by:

$$\left.\begin{aligned} q_{ib} &= (0,0,w_0),\\ T_b(z) &= T_0 + \Delta T f(z),\\ \mu_b(z) &= \frac{\mu_0}{1+\delta_T(T-T_0)-\delta_H(H-H_0)}\\ &= \frac{1}{1+f(z)V},\\ \rho_b(z) &= \rho_0\left[1-\alpha\Delta T f(z)\right],\\ M_b(z) &= M_0 - \left(\frac{k_l}{1+\chi_m}\right) f(z),\\ H_b(z) &= H_0 + \left(\frac{k_l}{1+\chi_m}\right) f(z). \end{aligned}\right\} \tag{9}$$

where, $f(z) = \dfrac{1}{1-e^{w_0 d/\kappa}}\left(e^{w_0 z/\kappa} - e^{w_0 d/\kappa}\right)$

and $V = \left(\delta_T - \dfrac{\delta_H \kappa_l}{1+\chi_m}\right)\Delta T.$

On the quiescent basic state finite amplitude perturbations are super imposed in the following form:

$q_i = q_{ib} + q_i'$, $p = p_b(z) + p'$, $T = T_b(z) + T'$, $\rho = \rho_b(z) + \rho_b'$, $H_i = H_{ib}(z) + H'$, $M_i = M_{ib}(z) + M'$ and $\mu = \mu_b + \mu'$, where, $q_i' = (u', v', w')$, p', T', $H_i' = (H_x', 0, H_z')$, $M_i' = (M_x', 0, M_z')$ and μ' are the perturbed quantities. Under the Boussinesq approximation, by the classical procedure of linear stability analysis, taking $t' = (d^2/\kappa)t^*$, $q_i' = (\kappa/d)q_i^*$, $p' = (\mu\kappa/d^2)p^*$, $T' = (\Delta T)T^*$ and $\phi' = (\kappa(\Delta T)d^2/1+\chi_m)\phi^*$, the dimensionless equations governing perturbations superposed over the quiescent basic state after dropping the primes and asterisks, can be written as

TABLE II
BOUNDARY COMBINATIONS AND CORRESPONDING TRIAL FUNCTIONS FOR RAYLEIGH-BÉNARD FERROCONVECTION

Case	Boundary	Boundary condition(BC)	Acronym	Trial functions
1	z = 0	$w = Dw = 0$ Rigid		$w_1 = 2z^4 - 5z^3 + 3z^2$
		$T = D\phi = 0$ Isothermal	RIFI	$T_1 = z^2 - z$
	z = 1	$w = D^2w = 0$ Free		$\phi_1 = cos(\pi z)$
		$T = D\phi = 0$ Isothermal		
2	z = 0	$w = D^2w = 0$ Free		$w_1 = z^4 - 2z^3 + z$
		$T = D\phi = 0$ Isothermal	FIFI	$T_1 = z^2 - z$
	z = 1	$w = D^2w = 0$ Free		$\phi_1 = cos(\pi z)$
		$T = D\phi = 0$ Isothermal		
3	z = 0	$w = Dw = 0$ Rigid		$w_1 = z^4 - 2z^3 + z^2$
		$T = D\phi = 0$ Isothermal	RIRI	$T_1 = z^2 - z$
	z = 1	$w = Dw = 0$ Rigid		$\phi_1 = cos(\pi z)$
		$T = D\phi = 0$ Isothermal		
4	z = 0	$w = Dw = 0$ Rigid		$w_1 = 2z^4 - 5z^3 + 3z^2$
		$DT = D\phi - T = 0$ Adiabatic	RAFI	$T_1 = z^2 - 1$
	z = 1	$w = D^2w = 0$ Free		$\phi_1 = \frac{z^2}{2} - z$
		$T = D\phi = 0$ Isothermal		
5	z = 0	$w = Dw = 0$ Rigid		$w_1 = z^4 - 2z^3 + z^2$
		$T = D\phi = 0$ Isothermal	RIFA	$T_1 = z^2 - 2z$
	z = 1	$w = D^2w = 0$ Free		$\phi_1 = \frac{-z^2}{2}$
		$DT = D\phi - T = 0$ Adiabatic		
6	z = 0	$w = D^2w = 0$ Free		$w_1 = z^4 - 2z^3 + z$
		$T = D\phi = 0$ Isothermal	FIFA	$T_1 = z^2 - 1$
	z = 1	$w = D^2w = 0$ Free		$\phi_1 = \frac{z^2}{2} - z$
		$DT = D\phi - T = 0$ Adiabatic		
7	z = 0	$w = Dw = 0$ Rigid		$w_1 = 2z^4 - 5z^3 + 3z^2$
		$T = D\phi = 0$ Isothermal	RIRA	$T_1 = z^2 - 2z$
	z = 1	$w = Dw = 0$ Rigid		$\phi_1 = \frac{-z^2}{2}$
		$DT = D\phi - T = 0$ Adiabatic		
8	z = 0	$w = D^2w = 0$ Free		$w_1 = z^4 - 2z^3 + z$
		$T = D\phi = 0$ Isothermal	FIRA	$T_1 = z^2 - 2z$
	z = 1	$w = Dw = 0$ Rigid		$\phi_1 = \frac{-z^2}{2}$
		$DT = D\phi - T = 0$ Adiabatic		
9	z = 0	$w = D^2w = 0$ Free		$w_1 = z^4 - \frac{3}{2}z^3 + \frac{1}{2}z$
		$DT = D\phi - T = 0$ Adiabatic	FARI	$T_1 = z^2 - 2z$
	z = 1	$w = Dw = 0$ Rigid		$\phi_1 = \frac{-z^2}{2}$
		$T = D\phi = 0$ Isothermal		

$$\left(\frac{1}{Pr}\frac{\partial}{\partial t}+\frac{Pe}{Pr}\frac{\partial}{\partial z}\right)\left(\nabla^2 w\right)=g_1(z)\nabla^4 w$$
$$+2\frac{\partial}{\partial z}\left(g_1(z)\right)\nabla^2\left(\frac{\partial w}{\partial z}\right)+\frac{\partial^2}{\partial z^2}\left(g_1(z)\right)\left(\frac{\partial^2 w}{\partial z^2}-\nabla_1^2 w\right) \tag{10}$$
$$+RM_1\frac{\partial}{\partial z}\left(f(z)\right)\left(\frac{\partial}{\partial z}\left(\nabla_1^2\phi\right)-\nabla_1^2 T\right)+R\nabla_1^2 T$$

$$\frac{\partial T}{\partial t}+g_2(z)w+Pe\frac{\partial T}{\partial z}=\nabla^2 T \tag{11}$$

$$M_3\nabla_1^2\phi+\frac{\partial^2\phi}{\partial z^2}-\frac{\partial T}{\partial z}=0 \tag{12}$$

where,

$$g_1(z)=\frac{1}{1+f(z)V},$$
$$g_2(z)=\frac{(Pe)e^{Pez}}{1-e^{Pe}},$$
$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}=\nabla_1^2+\frac{\partial^2}{\partial z^2}.$$

where (w,T,ϕ) are the dimensionless perturbations of the velocity, the temperature, and the magnetic potential respectively. The parameters appearing in (10)-(12) are:

$R=\dfrac{\alpha\rho_0 g\Delta T d^3}{\mu_0\kappa}$ (Rayleigh number),

$Pe=\dfrac{w_0 d}{\kappa}$ (throughflow Peclet number),

$Pr=\dfrac{\mu_0}{\rho_0\kappa}$ (Prandtl number),

$M_1=\dfrac{\mu_0 k_l^2\Delta T}{\rho_0 g\alpha(1+\chi_m)d}$ (buoyancy magnetic number),

$M_3=\dfrac{(1+M_0/H_0)}{(1+\chi_m)}$ (non-buoyancy magnetic number),

$V=\left(\delta_T-\dfrac{\delta_H\kappa_l}{1+\chi_m}\right)\Delta T$ (variable viscosity parameter).

C. Linear Stability Analysis

The perturbations for stationary convection are assumed to be periodic waves, employing the normal mode solution for (10)-(12) in the form:

$$\begin{aligned}w(x,y,z,t)&=w(z)e^{i(lx+my)},\\T(x,y,z,t)&=T(z)e^{i(lx+my)},\\\phi(x,y,z,t)&=\phi(z)e^{i(lx+my)}.\end{aligned} \tag{13}$$

we get the equations governing $w(z)$, $T(z)$ and $\phi(z)$, the amplitudes of perturbations of velocity, temperature and magnetic potential respectively. In (13) l and m are horizontal components of wave numbers in the x and y directions with $a^2=l^2+m^2$. Using $D=\frac{d}{dz}$ and incorporating (13) in (10)-(12), we get the following governing equations:

$$\left(g_1(z)(D^2-a^2)^2\right)w+D^2g_1(z)\left(D^2w+a^2w\right)$$
$$+RM_1a^2Df(z)\left(T-D\phi\right)-Ra^2T$$
$$+\left(D^3-a^2D\right)\left(2Dg_1(z)-\frac{Pe}{Pr}\right)w=0, \tag{14}$$

$$\left(D^2-a^2\right)T-g_2(z)w-PeDT=0, \tag{15}$$

$$\left(D^2-M_3a^2\right)\phi-DT=0. \tag{16}$$

D. Galerkin Method

The critical value of Rayleigh number as well as the wave number are computed using a combination of the Galerkin technique and shooting technique. We first explain the procedure of single-term Galerkin method for determining the expression for Rayleigh number explicitly. Towards this end we select $w(z)$, $T(z)$ and $\phi(z)$ in the form:

$$\begin{aligned}w(z)&=Aw_1(z),\\T(z)&=BT_1(z),\\\phi(z)&=C\phi_1(z).\end{aligned} \tag{17}$$

where A, B and C are constants, w_1, ϕ_1 and T_1 selected satisfy the respective boundary conditions as per Table II. The detailed derivation of boundary conditions for the nine boundary combinations are available in [26], [33]-[35].

The expression for the Rayleigh number is obtained by integrating (14)-(16) with respect to z between z = 0, z = 1 after multiplying by w, T and ϕ respectively and using (17) in the resulting equation and then using the condition for non-trivial solution of the resulting homogeneous equations in A, B and C, we get:

$$R=\frac{N_1\left(X_9-M_3a^2X_{10}\right)\left(X_7-PeX_{81}\right)}{\left(N_2N_3X_8+M_1a^2X_5X_8X_{11}\right)}. \tag{18}$$

where

$$\begin{aligned}N_1&=\left(X_1+X_2+X_3-\frac{Pe}{Pr}X_{21}\right),\\N_2&=\left(M_1a^2X_4-a^2X_6\right),\\N_3&=\left(M_3a^2X_{10}-X_9\right),\\X_1&=\langle g_1w_1\left(D^2-a^2\right)^2w_1\rangle,\\X_2&=2\langle w_1\left(D^2-a^2\right)Dw_1Dg_1\rangle,\\X_{21}&=2\langle w_1\left(D^3-a^2D\right)w_1\rangle,\\X_3&=\langle w_1\left(D^2+a^2\right)w_1D^2g_1\rangle,\\X_4&=\langle w_1\left(Df(z)\right)T_1\rangle,\\X_5&=\langle w_1\left(Df(z)\right)\left(D\phi_1\right)\rangle,\\X_6&=\langle w_1T_1\rangle,\quad X_7=\langle T_1\left(D^2-a^2\right)T_1\rangle,\\X_8&=\langle g_2(z)T_1w_1\rangle,\quad X_{81}=\langle T_1DT_1\rangle,\\X_9&=\langle\phi_1D^2\phi_1\rangle,\quad X_{10}=\langle\phi_1^2\rangle,\\X_{11}&=\langle\phi_1DT_1\rangle.\end{aligned}$$

E. Numerical Solution

The set of coupled equations (14)-(16) subject to the respective boundary conditions is solved numerically using the shooting method which is based on the Runge-Kutta-Fehlberg45 (RKF45) and Newton-Raphson methods and which gives an accurate eigenvalue. In this method the system of differential equations is transformed to a nine first order differential equations. To solve this system we require nine initial conditions but we have only four initial conditions and we need five more initial conditions to be prescribed. It is more important to choose appropriate initial conditions which can be improved by Newton-Raphson method. The solution process is repeated until the desired degree of accuracy is obtained. The detailed study of shooting method for eigen boundary value problem in convection is available in [34], [35].

III. Results and Discussion

At the onset of thermal convection the effect of vertical throughflow in temperature and magnetic field sensitive Newtonian ferromagnetic liquid is studied. Using the combination of Galerkin and shooting technique the critical eigenvalues for stationary convection are obtained. It is observed that the principle of exchange of stabilities is valid. Figs. 2-13 are the results obtained using shooting technique. Figs. 2 and 3 are respective plots of R_c and a_c versus V for different values of Pe and M_1 for FIFI boundary combination. From the graph we observe that the effect of increase in the value of V is to decrease R_c both in absence/presence of throughflow. This means that the effect of variable viscosity parameter is to destabilize the system. Also, it is evident from the graph that a_c decreases with increasing V suggesting the enlargement of the cell size. As M_1 increases R_c decreases and a_c increases, meaning that M_1 has a destabilizing influence on the onset of ferroconvection. The effect of nonlinearity of M_3 is found to have no influence on the stability of the system [1]. Figs. 4 and 5 are respective plots of R_c and a_c versus Pe for different values of V and M_1 for FIFI boundary combination. From the graph we observe that increase in the value of Pe is to increase R_c and this implies stabilization of the system. Further, a_c increases with increase in Pe. Figs. 6-9 and Figs. 10-13 are respective plots of RIRI and RIRA boundary combinations that correspond to Figs. 2-5 of FIFI. The various parameters influence on R_c and a_c are identical in all 3 boundary combination. Similar results are observed for remaining boundary combinations and not explicitly included here for want of space. We have also observed that our results are in good agreement for the limiting case of classical liquids in the absence of V with no magnetic field and no throughflow effect (see Table III). From Table IV, we observe that the Pr has insignificant influence on the onset of ferroconvection. The parameter V may take both negative and positive values. Positive values indicate temperature dominance over magnetic field in their influence on viscosity. Negative values indicate magnetic field dominance. From Figs. 14 and 15 it is clear that R_c and a_c decrease with increasing V for both negative and positive values of V.

A. General Results

By comparing the results on critical values, R_c and a_c, on the Rayleigh-Bénard convection in ferromagnetic liquids with vertical throughflow for respective different boundary combinations, we observe that the following is true:

$M_1 = 10$:
$R_c^{RIRI} > R_c^{RIRA} > R_c^{RIFI} > R_c^{RAFI} > R_c^{RIFA} > R_c^{FIRA} > R_c^{FIFI} > R_c^{FAFI} > R_c^{FIFA}$.

$a_c^{RIRI} > a_c^{RIRA} > a_c^{RIFI} > a_c^{FIRA} > a_c^{FIFI} > a_c^{RAFI} > a_c^{RIFA} > a_c^{FAFI} > a_c^{FIFA}$.

$M_1 = 20$:
$R_c^{RIRI} > R_c^{RIRA} > R_c^{RIFI} > R_c^{RIFA} > R_c^{RAFI} > R_c^{FIRA} > R_c^{FIFI} > R_c^{FAFI} > R_c^{FIFA}$.

$a_c^{RIRI} > a_c^{RIRA} > a_c^{RIFI} > a_c^{FIRA} > a_c^{RAFI} > a_c^{FIFI} > a_c^{RIFA} > a_c^{FAFI} > a_c^{FIFA}$.

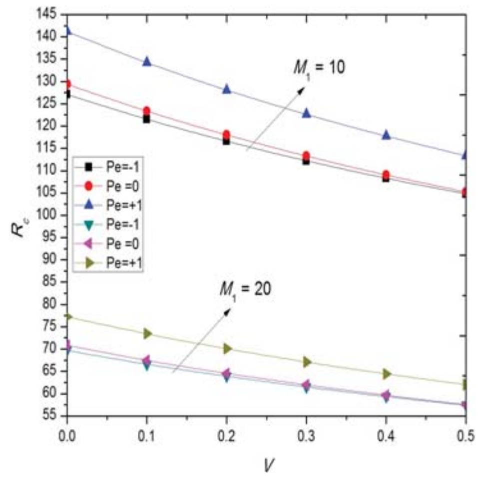

Fig. 2 Plot of R_c versus V for Different Values of Pe and M_1 with $M_3 = 1$ and $Pr = 10$ for the Boundary Combination FIFI

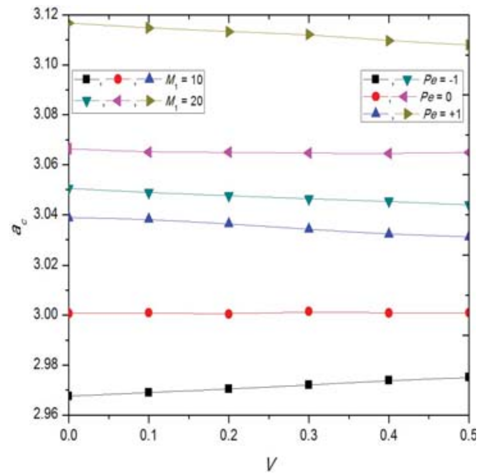

Fig. 3 Plot of a_c versus V for Different Values of Pe and M_1 with $M_3 = 1$ and $Pr = 10$ for the Boundary Combination FIFI

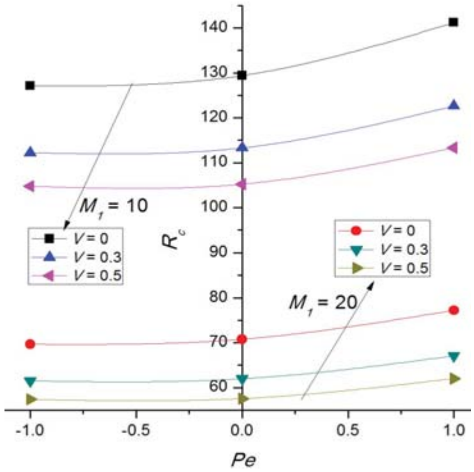

Fig. 4 Plot of R_c versus Pe for different values of V and M_1 with $M_3 = 1$ and $Pr = 10$ for the boundary combination FIFI

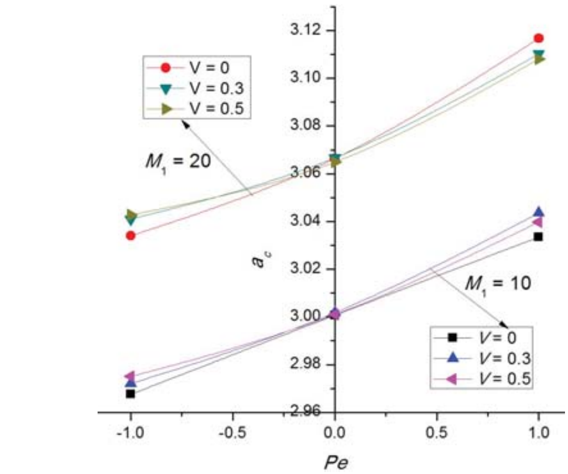

Fig. 5 Plot of a_c versus Pe for different values of V and M_1 with $M_3 = 1$ and $Pr = 10$ for the boundary combination FIFI

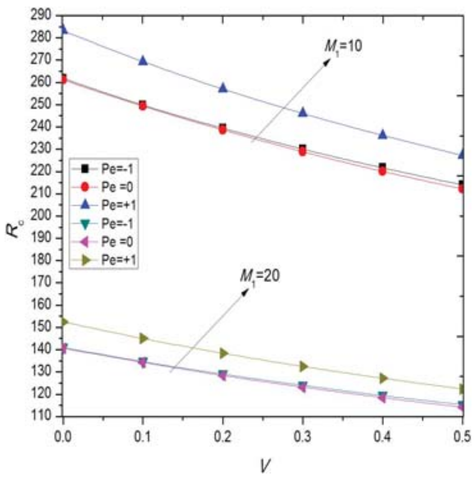

Fig. 6 Plot of R_c versus V for different values of Pe and M_1 with $M_3 = 1$ and $Pr = 10$ for the boundary combination RIRI

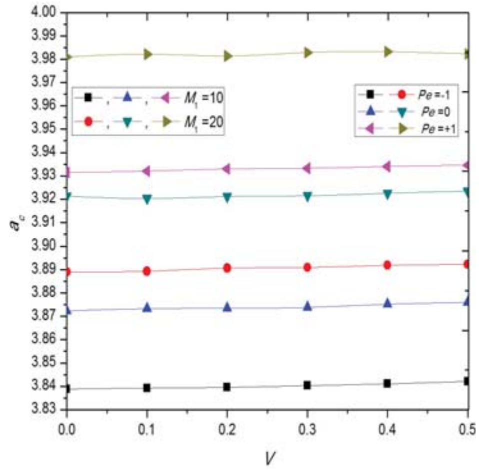

Fig. 7 Plot of a_c versus V for different values of Pe and M_1 with $M_3 = 1$ and $Pr = 10$ for the boundary combination RIRI

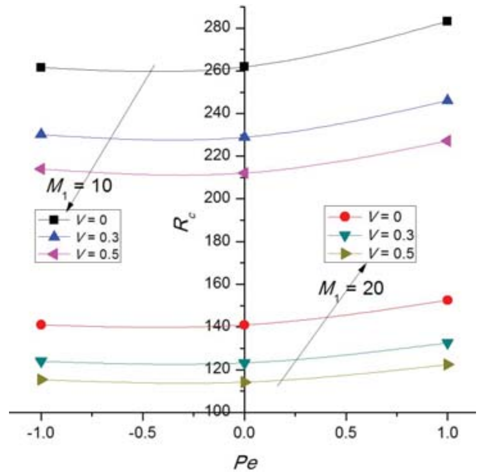

Fig. 8 Plot of R_c versus Pe for different values of V and M_1 with $M_3 = 1$ and $Pr = 10$ for the boundary combination RIRI

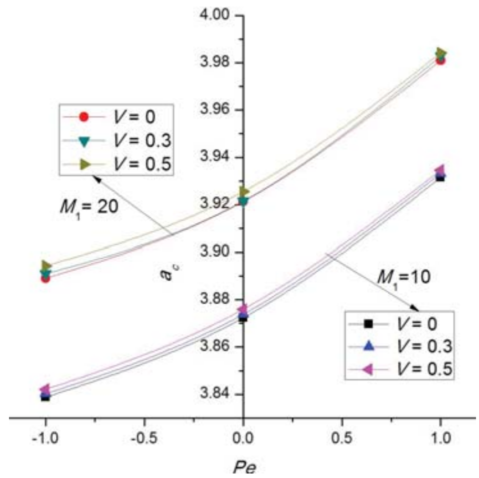

Fig. 9 Plot of a_c versus Pe for different values of V and M_1 with $M_3 = 1$ and $Pr = 10$ for the boundary combination RIRI

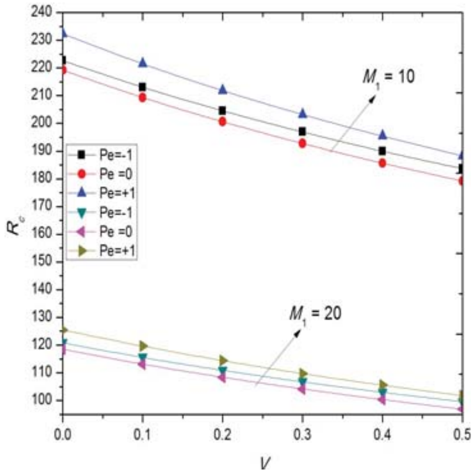

Fig. 10 Plot of R_c versus V for different values of Pe and M_1 with $M_3 = 1$ and $Pr = 10$ for the boundary combination RIRA

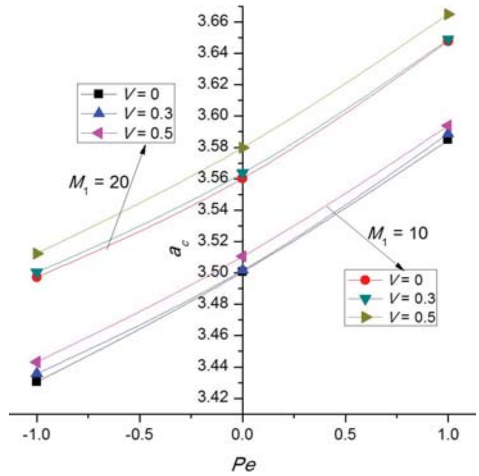

Fig. 13 Plot of a_c versus Pe for different values of V and M_1 with $M_3 = 1$ and $Pr = 10$ for the boundary combination RIRA

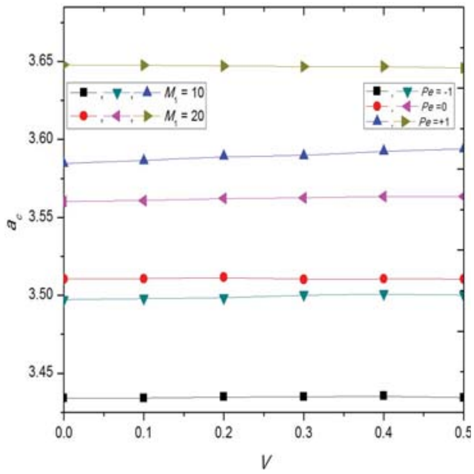

Fig. 11 Plot of a_c versus V for different values of Pe and M_1 with $M_3 = 1$ and $Pr = 10$ for the boundary combination RIRA

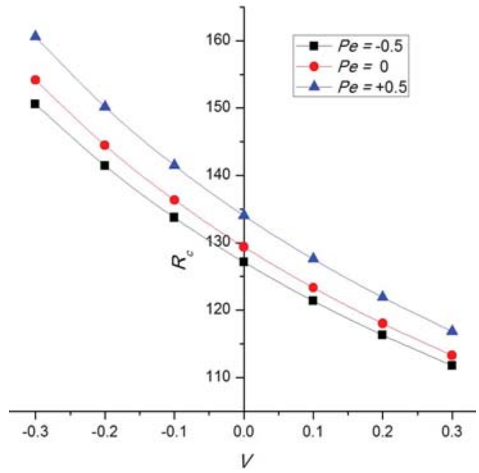

Fig. 14 Plot of R_c versus V for different values of Pe with $M_3 = 1$, $M_1 = 10$ and $Pr = 10$ for the boundary combination FIFI

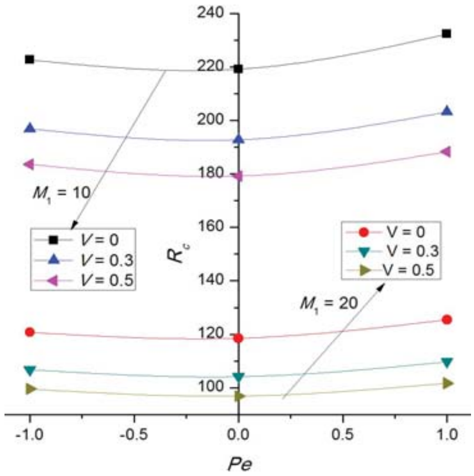

Fig. 12 Plot of R_c versus Pe for different values of V and M_1 with $M_3 = 1$ and $Pr = 10$ for the boundary combination RIRA

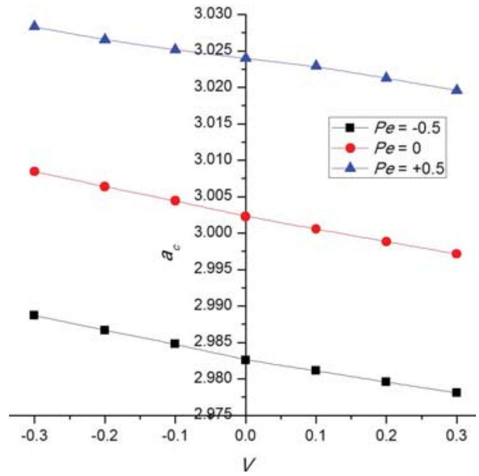

Fig. 15 Plot of a_c versus V for different values of Pe with $M_3 = 1$, $M_1 = 10$ and $Pr = 10$ for the boundary combination FIFI

TABLE III
COMPARISON OF R_c AND a_c OBTAINED BY DIFFERENT RESEARCHERS WITH THOSE OF PRESENT STUDY FOR THE LIMITING CASE $V = 0$ IN THE ABSENCE OF MAGNETIC FIELD AND THROUGH FLOW EFFECTS

BC	Platten and Legros [36]		Nield [18]		Present problem	
	R_c	a_c	R_c	a_c	R_c	a_c
RIFI	1100.657	2.68	1138	-	1100.65535	2.68225
RAFA	320	0	320	-	320.0000001	0.00001
RIRA	1295.781	2.55	1452	-	1295.78455	2.55185
RIFA	669.001	2.09	692	-	668.999579	2.08550
RAFI	816.748	2.21	953	-	816.747148	2.21465
FIFA	384.693	1.76	413	-	384.69325	1.7575

TABLE IV
VALUES OF R_c AND a_c FOR DIFFERENT VALUES OF Pr, V AND Pe

$M_3 = 1$, $M_1 = 10$ and $Pr = 10$

V	$Pe = -1$		$Pe = 0$		$Pe = +1$	
	R_c	a_c	R_c	a_c	R_c	a_c
0	128.752599	2.9966	129.390341	3.00215	141.220796	3.05045
0.1	122.770449	2.9972	123.34098	3.00095	134.225657	3.04955
0.2	117.505829	2.99675	118.018605	3.00065	128.086344	3.04625
0.3	112.824094	2.99705	113.286531	3.00155	122.638902	3.04355

$M_3 = 1$, $M_1 = 10$ and $Pr = 20$

V	$Pe = -1$		$Pe = 0$		$Pe = +1$	
	R_c	a_c	R_c	a_c	R_c	a_c
0	128.752131	2.9966	129.390341	3.00215	141.169942	3.0509
0.1	122.768935	2.9972	123.34098	3.00095	134.185338	3.0488
0.2	117.503358	2.99675	118.018605	3.00065	128.055542	3.0464
0.3	112.820691	2.9981	113.286531	3.00155	122.616846	3.0437

IV. CONCLUSION

The effect of V is to destabilize the system in presence of vertical through flow. The effect of increase in magnetic buoyancy number, M_1 is to destabilize the system. The nonlinearity of M_3 is found to have no influence on the onset of ferroconvection. The effect of increase in the value of Pr on the stability of the system is insignificant. The effect of through flow measured by the Peclet number, Pe is to delay the onset of ferroconvection and is independent of the direction of through flow.

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